Un analogue mésoscopique du “paradoxe de Braess” mis en évidence grâce aux sondes locales

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Outline

- What is “Braess paradox”?
- Introduction to Scanning-Gate Microscopy (SGM).
- Quantum simulations of a mesoscopic Braess paradox.
- SGM experiments and interpretation.
- Conclusion.

The original Braess paradox for road network

*Braess (1968): **Adding extra road** to a **congested network**, where the drivers can freely choose their route, can in **some cases** reduce overall performance, i.e. increase the travel time to all drivers.*

Interview of Cédric Villani (Fields medal in 2010) on France Info, 22 December 2011 + H. Youn *et al.*, *Phys. Rev. Lett.* **101**, 128701 (2008):

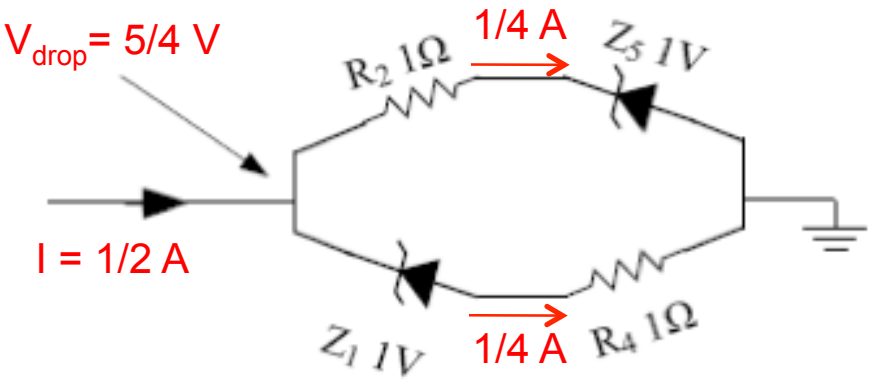
- Many examples of Braess paradox in complex networks of congested cities : Seoul, Boston, London, New-York, Stuttgart ….. **Nice** (tramway).

- In some cases, e.g. Seoul, closing a well-selected high-speed road counter-intuitively relieves congestion !!

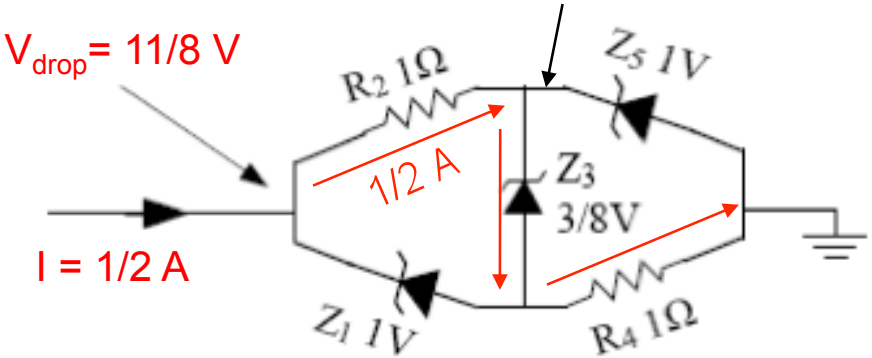
Theoretical framework: Game theory explains the original Braess paradox quite well.

The extension to other classical networks, e.g. electrical

See: Cohen & Horowitz, *Nature* (1991) and Penchina & Penchina, *Am. J. Phys.* (2003)



V=7/8V < 1V →
Z_5 and Z_1 are blocked



A Wheastone bridge with **non-linear** Zener diodes at constant injected current = 1/2 A →
At equilibrium, a 1/4 A current flows in each branch → voltage drop= 5/4 V.

An additional (lower-voltage) Zener bypasses the two branches →
At equilibrium, the entire current flows through R_2-Z_3-R_4 because the 1V Zener are blocked → voltage drop= 11/8 V > 5/4 V.

Braess paradox: Offering a new "zero-impedance" path to current increases impedance !

Note: If only linear components are used, no paradox occurs.

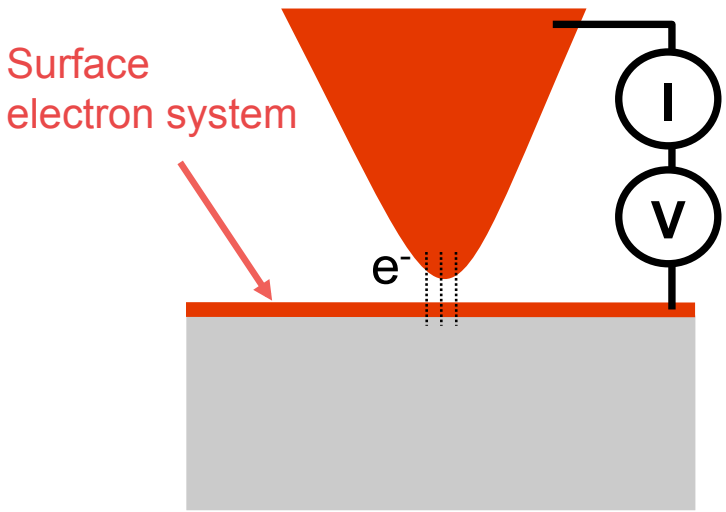
Known so far for classical networks only, we now have extended Braess's paradox to the quantum world thanks to Scanning-Gate Microscopy:

M.G. Pala *et al.*, *Phys. Rev. Lett.* **108**, 076802 (2012)

SGM versus STM

STM

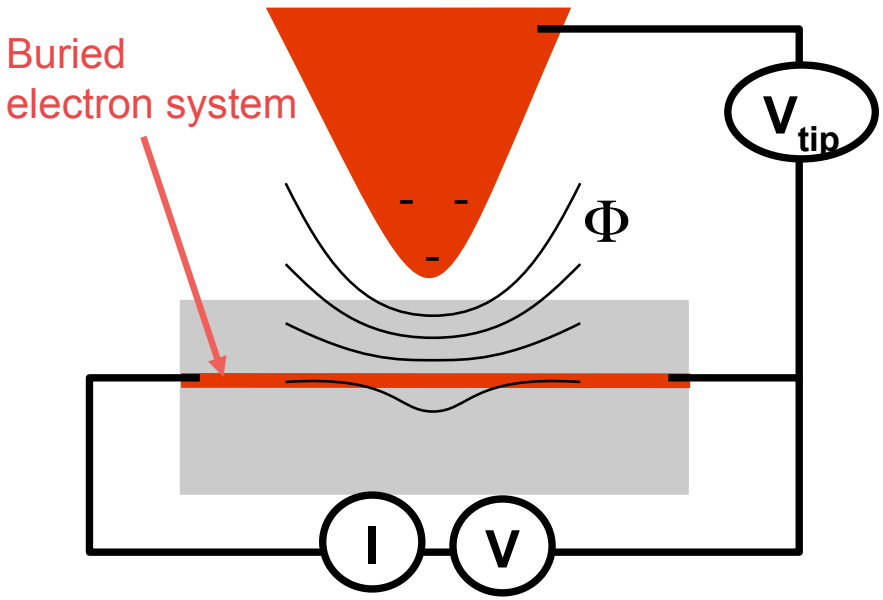
Scanning Tunneling Microscopy



- conducting surfaces
- tunnelling current
- interference effects & local density of states (LDOS)

SGM

Scanning Gate Microscopy



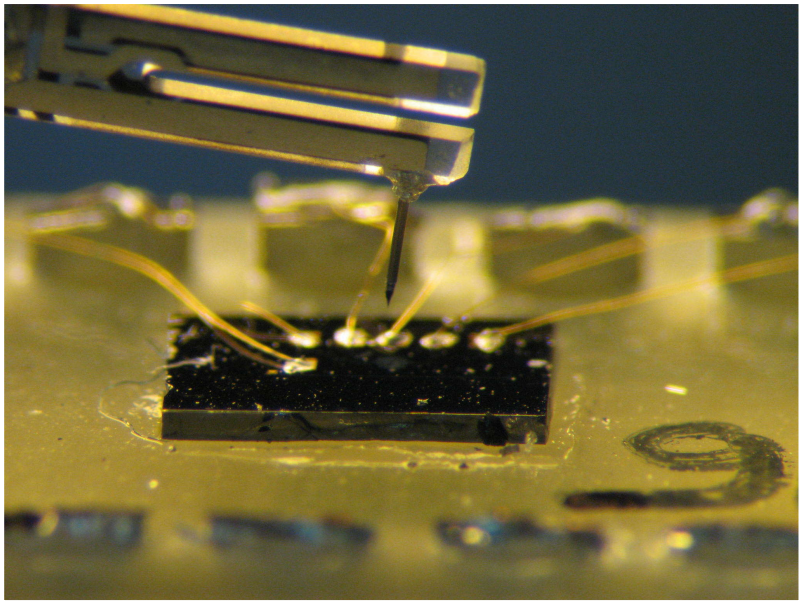
- insulating surfaces
- **buried** high mobility 2DEG heterostructure
- **device conductance** perturbed by local gate effects

→GOAL: local studies of electron systems not accessible to STM; locally perturb or control the electron transport.

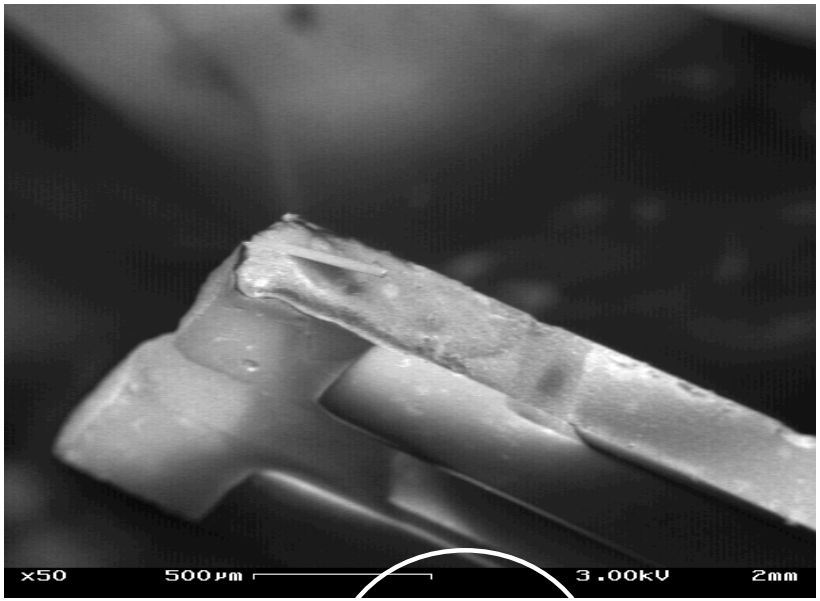
SGM probes

NO LIGHT ! → Use of piezoelectric quartz tuning fork

Karrai (1995) for NSOM → AFM, STM, SGM



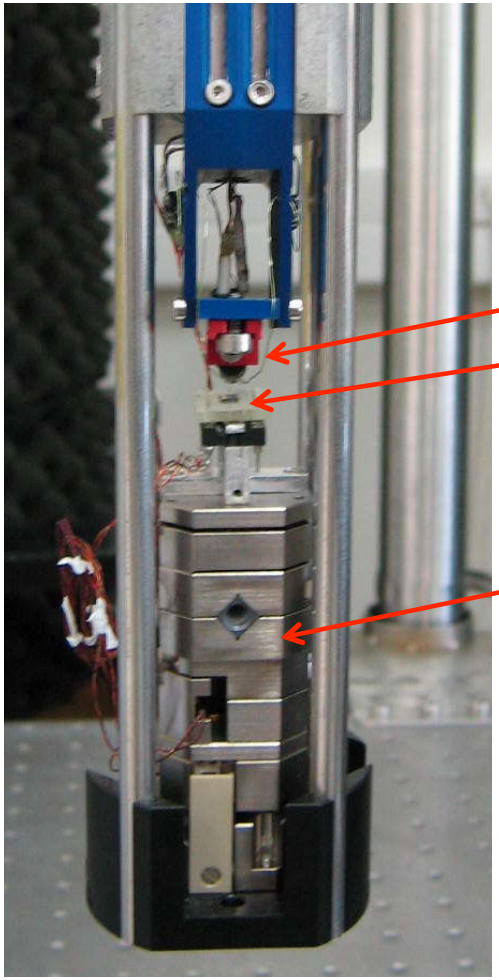
STM tip glued to one metallic pad of the tuning fork



Our favorite: conductive AFM cantilever glued to one metallic pad of the tuning fork

SGM microscope

Homemade AFM: low temp.=4 K and high magnetic field= 9 T.



sample

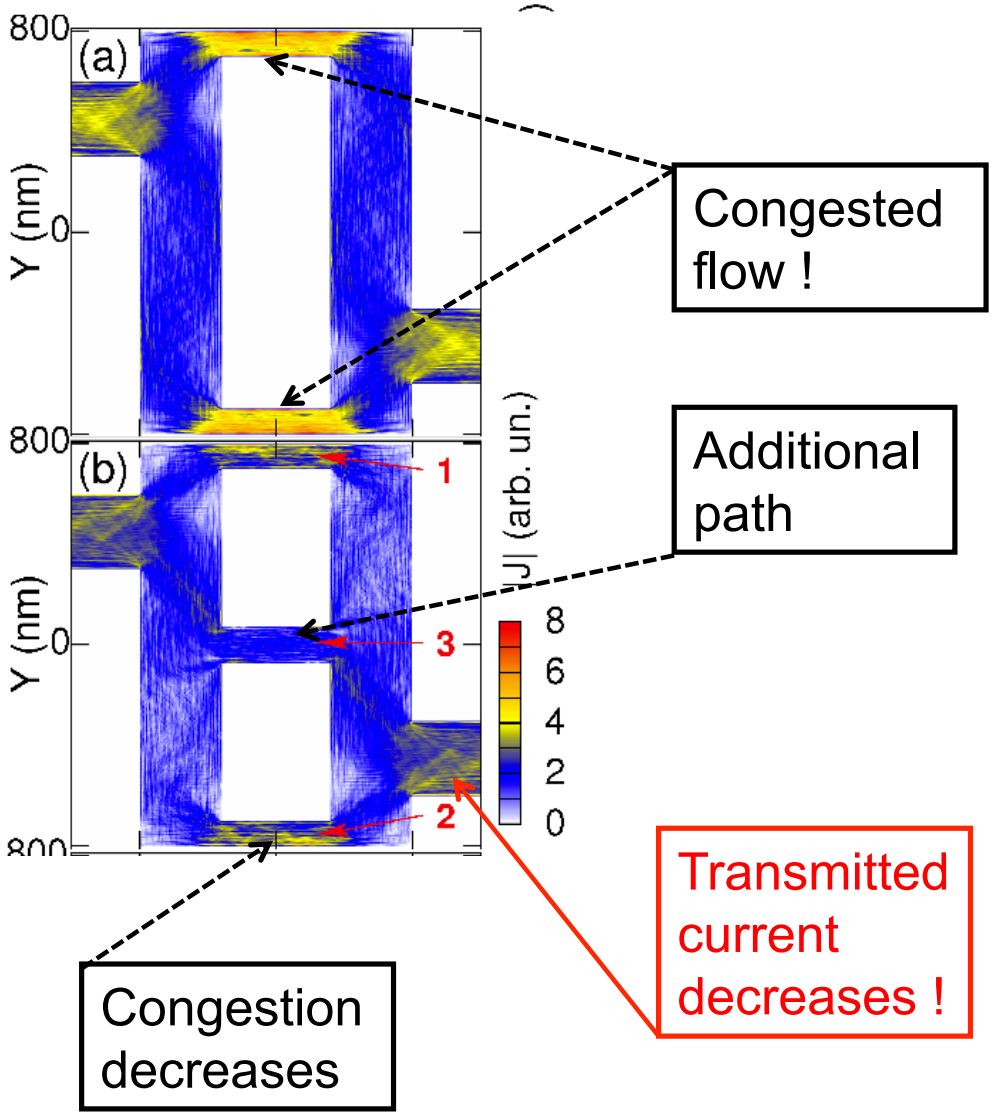
cryomotors
& scanner

Simulations of a rectangular mesoscopic network (I)

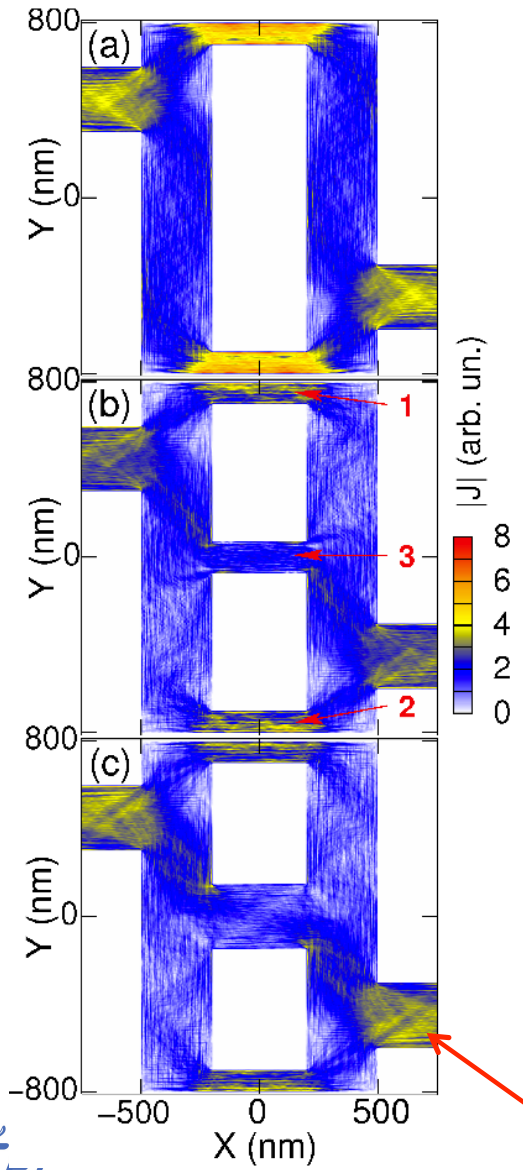
Quantum simulations of spatial distribution current density $|J|(X,Y)$ of:

✓ Rectangular corral with "mesoscopic" dimensions (smaller or comparable to mean-free path & coherence length at 4 K).

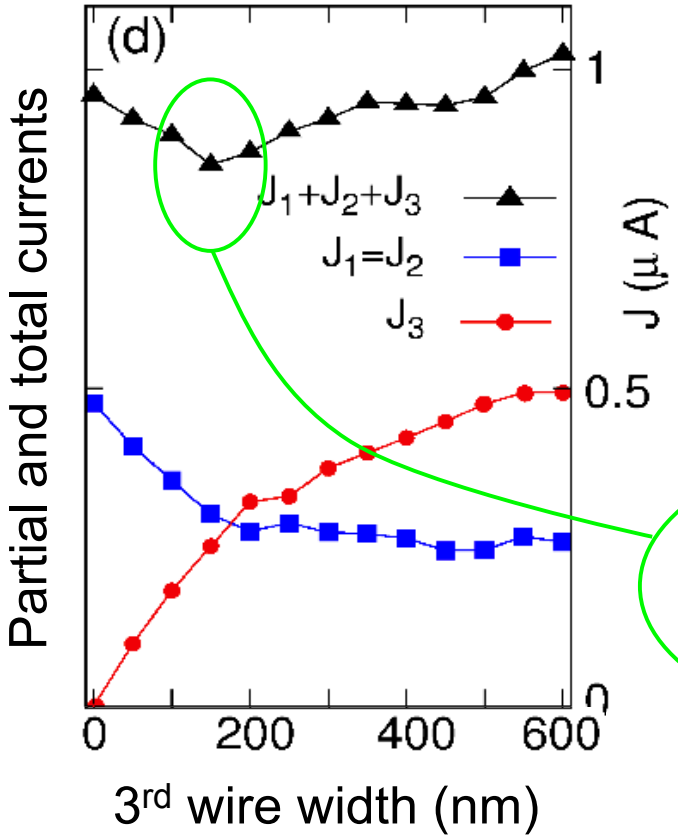
✓ Made of a GaInAs/AlInAs heterostructure: 2DEG of carrier density = 3.5 x 10^{11}cm^{-2} (Fermi wavelength around 40 nm) and mobility = 100000 cm^{2}V^{-1}s^{-1}.



Simulations of a rectangular mesoscopic network (II)



Partial and total currents



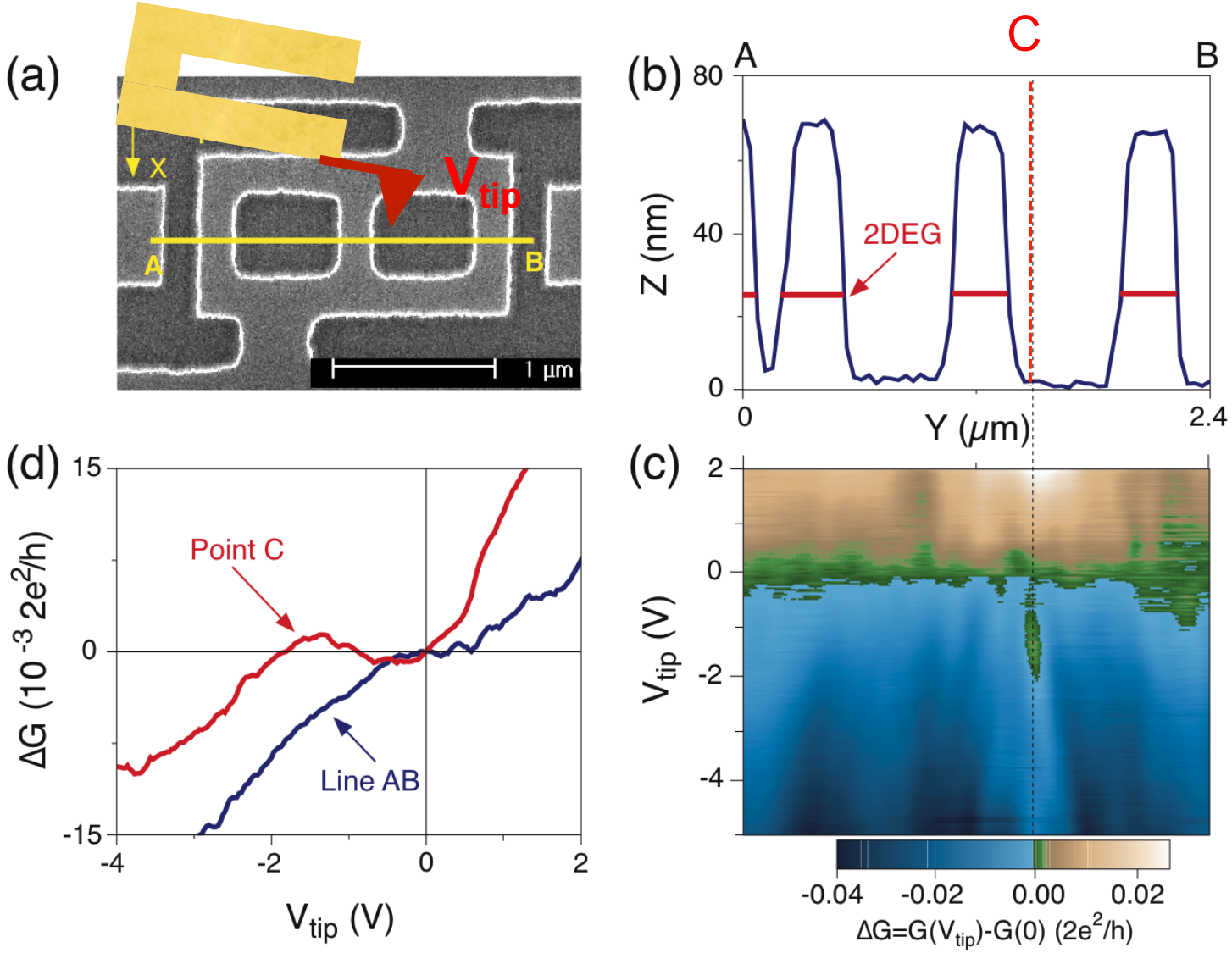
On adding a 3rd path, J_3 increases first but $J_1 + J_2$ decreases more rapidly

→ Total current drops:

BRAESS ?!

Current increases back

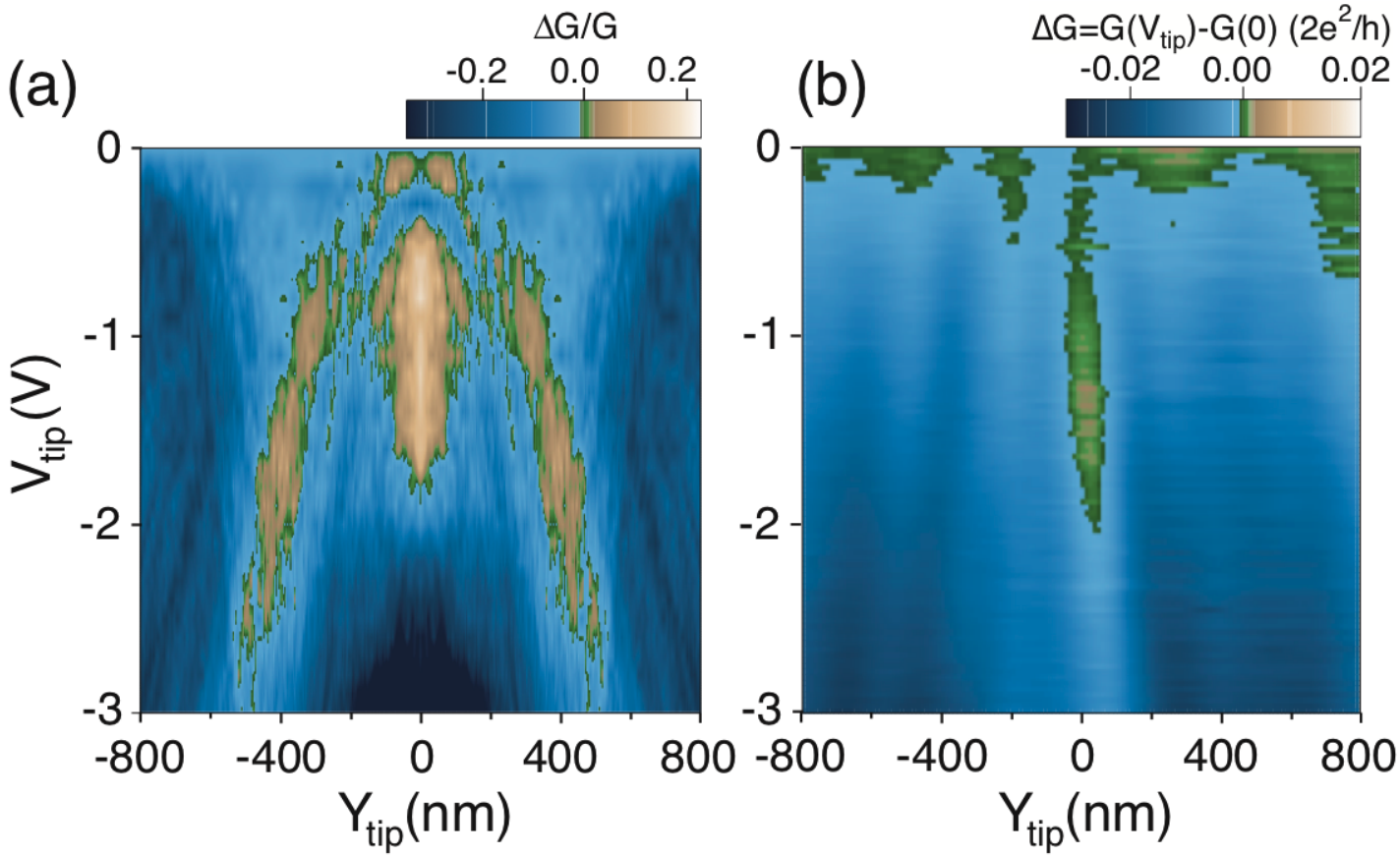
SGM experiments



Tapping Topo.

Non contact SGM

Counter-intuitive conductance increase

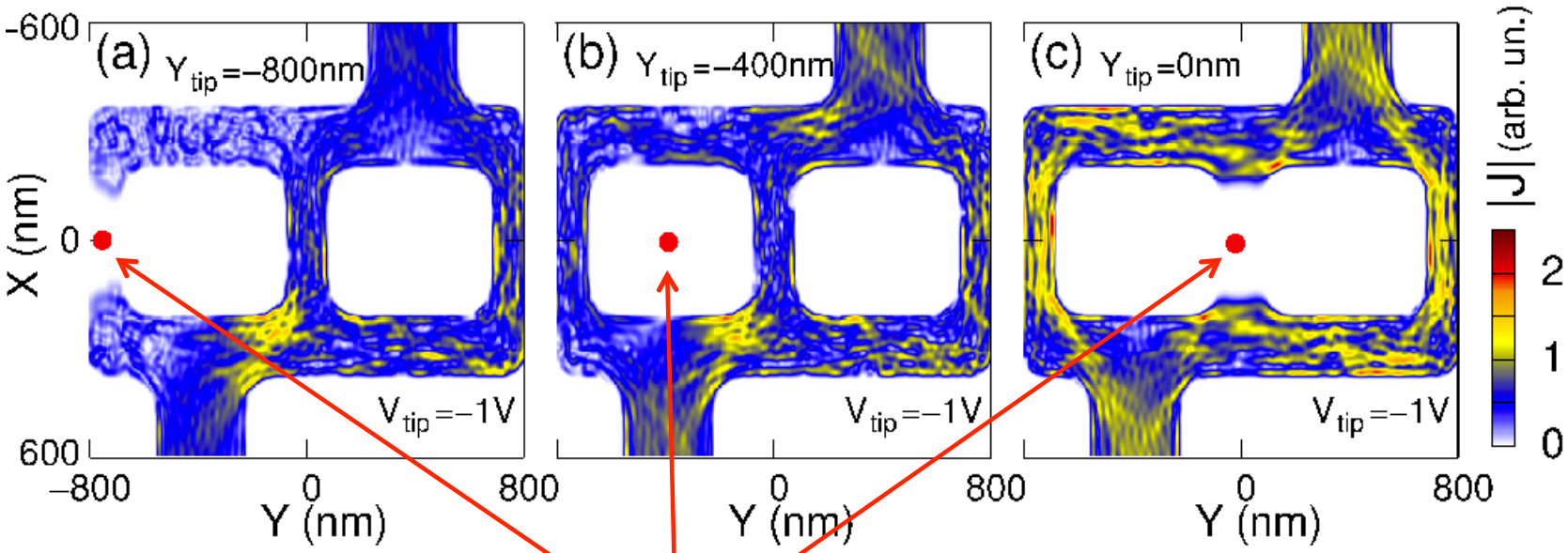


Simulation with tip included

Experiment

Insight into the microscopic behaviour (simulations)

$$|J|(X,Y)$$



Scanning large (fully depleting) tip potential

M.G. Pala *et al.*, *Phys. Rev. Lett.* **108**, 076802 (2012)

Conclusions

A mesoscopic Braess-like paradox has been discovered →
Many questions:

- What is the microscopic origin?
- How far the analogy with the classical paradox can be made?
- What is the best suited geometry?
- What's about a plasmonic Braess-like paradox?
- Towards new electronic devices?
- …..

MERCI !