Un analogue mésoscopique du “paradoxe de Braess” mis en évidence grâce aux sondes locales

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Outline

• What is “Braess paradox”?  
• Introduction to Scanning-Gate Microscopy (SGM).  
• Quantum simulations of a mesoscopic Braess paradox.  
• SGM experiments and interpretation.  
• Conclusion.
The original Braess paradox for road network

Braess (1968): Adding extra road to a congested network, where the drivers can freely choose their route, can in some cases reduce overall performance, i.e. increase the travel time to all drivers.


• In some cases, e.g. Seoul, closing a well-selected high-speed road counter-intuitively relieves congestion !!

Theoretical framework: Game theory explains the original Braess paradox quite well.
The extension to other classical networks, e.g. electrical

A Wheatstone bridge with non-linear Zener diodes at constant injected current = 1/2 A → At equilibrium, a 1/4 A current flows in each branch → voltage drop= 5/4 V.

An additional (lower-voltage) Zener bypasses the two branches → At equilibrium, the entire current flows through $R_2-Z_3-R_4$ because the 1V Zener are blocked → voltage drop = 11/8 V > 5/4 V.

Braess paradox: Offering a new “zero-impedance” path to current increases impedance!

Note: If only linear components are used, no paradox occurs.
Known so far for classical networks only, we now have extended Braess’s paradox to the quantum world thanks to Scanning-Gate Microscopy:

SGM versus STM

STM
Scanning Tunneling Microscopy

- conducting surfaces
- tunnelling current
- interference effects & local density of states (LDOS)

SGM
Scanning Gate Microscopy

- insulating surfaces
- **buried** high mobility 2DEG heterostructure
- **device conductance** perturbed by local gate effects

→ GOAL: local studies of electron systems not accessible to STM; locally perturb or control the electron transport.
SGM probes

NO LIGHT! → Use of piezoelectric quartz tuning fork
Karrai (1995) for NSOM → AFM, STM, SGM

STM tip glued to one metallic pad of the tuning fork

Our favorite: conductive AFM cantilever glued to one metallic pad of the tuning fork
Homemade AFM: low temp. = 4 K and high magnetic field = 9 T.
Simulations of a rectangular mesoscopic network (I)

Quantum simulations of spatial distribution current density $|J|(X,Y)$ of:

- Rectangular corral with "mesoscopic" dimensions (smaller or comparable to mean-free path & coherence length at 4 K).
- Made of a GaInAs/AlInAs heterostructure: 2DEG of carrier density $= 3.5 \times 10^{11}\text{cm}^{-2}$ (Fermi wavelength around 40 nm) and mobility $= 100000 \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

Congestion decreases!

Transmitted current decreases!
Simulations of a rectangular mesoscopic network (II)

On adding a 3rd path, $J_3$ increases first but $J_1 + J_2$ decreases more rapidly.

Partial and total currents

Current increases back

Total current drops:

BRAESS ?!
SGM experiments

Non contact SGM

Tapping Topo.

\( \Delta G = G(V_{\text{tip}}) - G(0) \) (2e^2/h)
Counter-intuitive conductance increase

Simulation with tip included

Experiment
Insight into the microscopic behaviour (simulations)

\[ |J|(X,Y) \]

Scanning large (fully depleting) tip potential

Conclusions

A mesoscopic Braess-like paradox has been discovered →

Many questions:

• What is the microscopic origin?

• How far the analogy with the classical paradox can be made?

• What is the best suited geometry?

• What’s about a plasmonic Braess-like paradox?

• Towards new electronic devices?

• …..
MERCI !